It is easy to show that the first-order solution that appears in Eq. (30) is given by

$$R_1 = R_1(\tau,\sigma,\epsilon)\exp(i\nu_0\xi) \tag{31}$$

and R_1 is calculated in a similar way as outlined in Sec. II.

From Eqs. (27-31), we notice two important results. First, the equations of motion that govern the bubbly flow are written in terms of τ and σ alone, which represent the slow temporal and spatial variations respectively. Thus, solving Eqs. (27) and (28) numerically, a larger time step can be used which is $1/\epsilon$ bigger than the time step needed for solving the original set of equations (18-20). Second, the fast oscillations affect the bulk motion of the two phases at least in order ϵ^2.

IV. Conclusions

The existence of two distinct time scales in bubbly flows was used in order to integrate over the fast variations that occur due to the natural volume oscillations. First, the oscillations of a single bubble in a slowly varying pressure field were investigated. Using the multiple-scale technique it was shown that these oscillations are enhanced if the liquid's pressure is growing. Then, a procedure was introduced that averages the equations of bubbly flows over the fast oscillations.

As a result, a set of reduced differential equations is obtained which depend only on the slowly varying independent variables. This procedure enables the use of large time steps in numerical computations.

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Optimization of Equivalent Periodic Truss Structures

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Introduction

SIMPLE beam models of complex periodic space structures can be effectively used to optimize the dimensions of the structural members. This could result in enormous cost savings as well as provide the analyst with valuable insights into the behavior of complex periodic space structures. However, care must be exercised when computing member loads from these simple beam models. Specifically, for a long cantilever periodic space structure, the fundamental frequency and tip displacement can be accurately determined using a Bernoulli-Euler beam. However, the member loads can be accurately calculated only by using a Timoshenko beam.

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Behavior of Truss Structure

The cantilever truss shown in Fig. 1 is taken to illustrate the difference obtained by using a Bernoulli-Euler beam and a Timoshenko beam. The member properties are as follows:

$$E = 71.7\times 10^9\ \mathrm{N/m^2} \qquad L = 75\ \mathrm{m}$$

$$\rho = 2768\ \mathrm{kg/m^3} \qquad A_c = 80\times 10^{-6}\ \mathrm{m^2}$$

$$L_c = 7.5\ \mathrm{m} \qquad A_g = 60\times 10^{-6}\ \mathrm{m^2}$$

$$L_g = 5.0\ \mathrm{m} \qquad A_d = 40\times 10^{-6}\ \mathrm{m^2}$$

where A_c, A_g, and A_d are the cross-sectional areas of the vertical, horizontal, and diagonal members, respectively. These dimensions and properties were chosen so that the truss could be considered a Bernoulli-Euler beam. One load condition, as shown in Fig. 1, is imposed: $P = 200$ N.

Using the approach developed by Sun et al.,[1] the equivalent beam properties can be obtained as follows:

$$AE = 2A_dE(\beta^3+\gamma) \tag{1}$$

$$EI = 0.5EA_cL_g^2 \tag{2}$$

$$KAG = 2A_dE\alpha^2(1-\alpha^2)^{1/2} \tag{3}$$

where

$$\alpha = L_g/L_d, \qquad \beta = L_c/L_d, \qquad \gamma = A_c/A_d$$

The design problem to be solved here can be stated as follows: Find the dimension of the members such that the mass of the structure

$$m = \{2n[A_cL_c + A_dL_d] + (n+1)[A_gL_g]\}\rho \tag{4}$$

where n, the number of periodic structures, is minimized subject to the following constraints:

1) Frequency Constraint

$$(\omega/\omega_0) - 1 \geq 0 \tag{5}$$

where ω is the fundamental frequency of a Bernoulli-Euler beam and is given by

$$\omega = [1.875/L]^2\sqrt{EIL/m_r} \tag{6}$$

and $m_r = m - A_gL_g\rho$ (since base girder fixed mass is not participating). Also, ω_0 is the minimum allowable fundamental frequency, which for this problem is 5 rad/s.

2) Tip Displacement Constraint

$$1 - (\Delta_x/\Delta_{max}) \geq 0 \tag{7}$$

where Δ_x is the tip displacement given by

$$\Delta_x = PL^3/3EI \qquad \text{(Bernoulli-Euler)}$$

$$\Delta_x = (PL^3/3EI) + (PL/KAG) \qquad \text{(Timoshenko)} \tag{8}$$

Also, Δ_{max} is the maximum allowable tip displacement, which for this problem is 0.4 m.

3) Column Buckling Constraint

$$1-(P_c/P_{cr})\geq 0 \quad (9)$$

where P_{cr} is the Euler buckling load for a pinned-pinned column and is given by

$$P_{cr}=\pi^2 EI_c/L_c^2 \quad (10)$$

where I_c is the area moment of inertia of the column, and P_c is the applied column load calculated by

$$P_c=\Delta_c A_c E/L_c \quad (11)$$

where Δ_c is the axial (Bernoulli-Euler and Timoshenko) deflection of the column given by

$$\Delta_c=(PL_g/4EI)\{L^2-x^2\} \quad (12)$$

where x is the distance from the tip of the truss to the column under analysis.

4) Diagonal Buckling Constraint

$$1-(P_d/P_{cr})\geq 0 \quad (13)$$

where P_{cr} is the Euler buckling load for a pinned-pinned diagonal and is given by

$$P_{cr}=\pi^2 EI_d/L_d^2 \quad (14)$$

where I_d is the area moment of inertia of the diagonal and is the applied diagonal load calculated by

$$P_d=\Delta_d A_d E/L_d \quad (15)$$

where Δ_d is the axial deflection of the diagonal given by

$$\Delta_d=|\Delta_c\cos\theta-\Delta_x\sin\theta| \quad (16)$$

where θ is the angle between the column and the diagonal and is given by

$$\theta=\tan^{-1}[L_g/L_c] \quad (17)$$

and Δ_c is the axial deflection of the column given by

$$\Delta_c=(PL_g/4EI)\{L^2-x^2\} \quad \text{(Bernoulli-Euler and Timoshenko)} \quad (18)$$

where x is the distance from the tip of the truss to the diagonal under analysis. Also, Δ_x is the transverse deflection of the truss at the diagonal and is given by

$$\Delta_x=(P/6EI)[2L^3-2x^3-3L^2x+3x^3] \quad \text{(Bernoulli-Euler)} \quad (19)$$

$$\Delta_x=(P/6EI)[2L^3-2x^3-3L^2x+3x^3] + (PL/KAG)[L^2-x^2] \quad \text{(Timoshenko)} \quad (20)$$

5) Girder Buckling Constraint

$$1-(P_g/P_{cr})\geq 0 \quad (21)$$

where P_{cr} is the Euler buckling load for a pinned-pinned girder and is given by

$$P_{cr}=\pi^2 EI_g/L_g^2 \quad (22)$$

where I_g is the area moment of inertia of the girder and P_g is the applied girder load, which is assumed to equal $P/2$ (girder sized to support applied load).

Design Algorithm

A general conjugate gradient algorithm and golden section acceptable point search are used to optimize the truss struc-

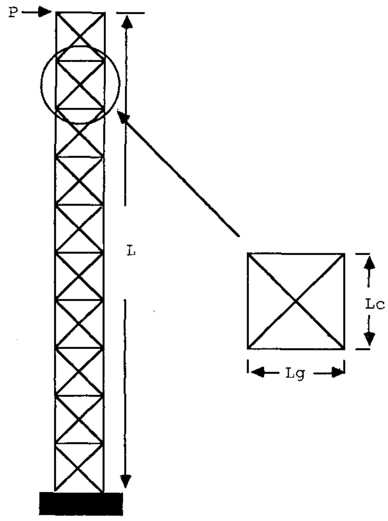

Fig. 1 Cantilever truss structure (10 bays).

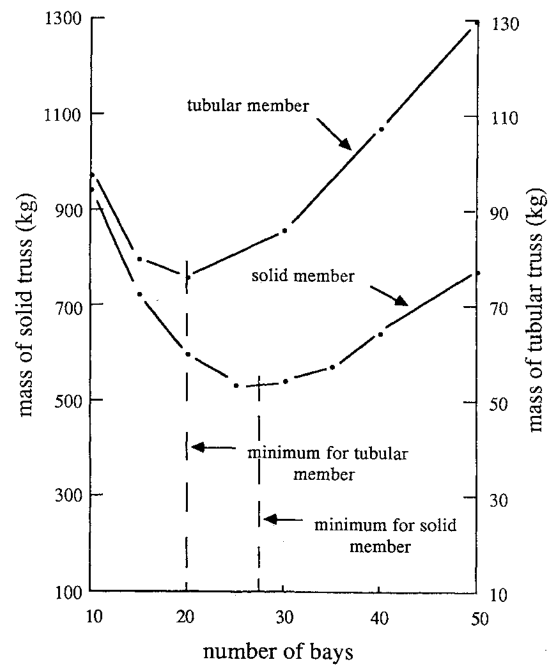

Fig. 2 Optimized mass of truss structure as function of number of bays.

Table 1 Results for a 10-bay truss

	Bernoulli-Euler	Timoshenko	MSC/NASTRAN
Eqs. (19) and (20), Δ_x, m	0.3922	0.4025	0.4015
Eq. (15), P_d, N	17.35	197.7	180.3

ture.[2] Two types of truss member cross sections are considered: a solid rod and tubular member. In addition to the existing constraints, manufacturing constraints are imposed on these members, given by

$$(r/r_{\min}) - 1 \geq 0 \tag{23}$$

$$(t/t_{\min}) - 1 \geq 0 \tag{24}$$

where

$r_{\min}$ = minimum outside radius of member = 0.001 m

$t_{\min}$ = minimum thickness of tubular member = 0.0004 m

Comparison of Bernoulli-Euler and Timoshenko Beam Approach

During the optimization process, the dominant constraint conditions were frequency constraint, column buckling constraint (solid members), diagonal buckling constraint, tip displacement constraint (tubular members), and minimum thickness constraint. For all of the constraint conditions except the diagonal buckling constraint, the Bernoulli-Euler beam gives almost identical results to the Timoshenko beam. However, for the critical diagonal member (adjacent to the centilevered end of the truss), the internal forces calculated using a Bernoulli-Euler beam are totally erroneous.

As an example, for a 10-bay truss, the tip deflections and diagonal member load computed using a Bernoulli-Euler and Timoshenko beam are compared to those determined using a MSC/NASTRAN truss model. The results are presented in Table 1. It is apparent that the Bernoulli-Euler beam predicts accurate tip deflections and yet totally erroneous diagonal member loads.

Comparison of Structural Efficiency

The optimized masses of the tubular member truss and the solid member truss are compared as a function of number of bays in Fig. 2. For the optimized structure, the tubular truss is quite attractive from a mass consideration as well as from the reduction in the number of bays required.

Conclusions

Through the use of equivalent modeling and simple hand calculations, an optimization of a truss structure was achieved. In addition, valuable insight was gained into the relative performance of a tubular and solid member truss structure. This approach could be extended to more complicated structures, resulting in inexpensive preliminary optimization. However, the analyst must be careful in selecting his equivalent models in order to avoid erroneous results.

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Perfect Gas Effects in Compressible Rapid Distortion Theory

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Introduction

THE interaction of unsteady disturbances with aerodynamic flows is a problem of fundamental interest, with applications to sound generation, structural vibration, and aeroelastic flutter. In the classical treatment of such problems, the mean flow is generally assumed to be uniform. However, in actual applications the mean flow gradients are often substantial. A more realistic approach is to allow the mean flow to be nonuniform, but to linearize the unsteady disturbances about this nontrivial mean flow. The analysis of linear, inviscid disturbances to irrotational mean flows has come to be known as Rapid Distortion Theory.

Hunt[1] considered the interaction of upstream vorticity disturbances with the incompressible flow around a two-dimensional body. He first determined the vorticity field and then integrated to find the induced velocities. An additional irrotational disturbance was required to satisfy the no-flow condition on the body surface. Goldstein[2] considered compressible flow and developed a simpler formulation (discussed below), which requires the solution of a single inhomogeneous wave equation.

Goldstein's inhomogeneous wave equation is linear but has variable coefficients and a complicated source term. The variable coefficients are functions of the compressible mean flow, which in general must be determined numerically. Kerschen and Balsa[3] showed that introduction of the tangent gas relations[4] allows the inhomogeneous wave equation to be transformed into a much simpler form. With the further assumption that the mean flow is a small disturbance to a uniform flow, analytical expressions could be given for the variable coefficients and source term. Unfortunately, we have found in later work that the tangent gas approximation leads to erroneous results in certain situations, as in the example included here of high-frequency gusts interacting with an airfoil. Essentially, the tangent gas approximation leads to unrealistic estimates of the variation in the speed of sound. The main purpose of this Note is to present an alternative simplified form of Goldstein's inhomogeneous wave equation that incorporates the perfect gas thermodynamic relations.

Analysis

We consider small-amplitude disturbances to a steady, irrotational, compressible mean flow. The mean flow is assumed to be uniform far upstream, and convected vortical and entropic disturbances are imposed on this uniform flow U_∞. Linearizing the equations of motion about the mean flow, and neglecting viscous and heat-conduction effects, Goldstein showed that the perturbations are described by the following equations:

$$\boldsymbol{u}' = \nabla G' + \boldsymbol{v}' \tag{1}$$

$$\frac{\mathrm{D}_0}{\mathrm{D}t}\left(\frac{1}{a_0^2}\frac{\mathrm{D}_0 G'}{\mathrm{D}t}\right) - \frac{1}{\rho_0}\nabla\cdot(\rho_0 \nabla G') = \frac{1}{\rho_0}\nabla\cdot(\rho_0 \boldsymbol{v}') \tag{2}$$

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